Consider the curve $\vec{r}(t) = < 2\sin 2t, 3t, 2\cos 2t >$.

SCORE: _____ / 55 PTS

[a] Find the unit tangent vector $\vec{T}(t)$.

$\vec{r}\,'(t) = \langle 4\cos 2t, 3, -4\sin 2t \rangle$

$\|\vec{r}\,'(t)\| = \sqrt{16\cos^2 2t + 9 + 16\sin^2 2t} = \sqrt{16+9} = 5$

$\vec{T}(t) = \frac{1}{5}\langle 4\cos 2t, 3, -4\sin 2t \rangle$

[b] Find parametric equations for the tangent line to the curve at the point $(\sqrt{3}, \pi, -1)$.

$3t = \pi \longrightarrow t = \frac{\pi}{3}$

$\vec{r}\,'(\frac{\pi}{3}) = \langle 4\cos\frac{2\pi}{3}, 3, -4\sin\frac{2\pi}{3} \rangle$

$= \langle -2, 3, -2\sqrt{3} \rangle$

$\vec{l}(t) = \langle \sqrt{3}, \pi, -1 \rangle + t\langle -2, 3, -2\sqrt{3} \rangle$

$x = \sqrt{3} - 2t$

$y = \pi + 3t$

$z = -1 - 2\sqrt{3}t$

[c] Find $\vec{T}\,'(t)$.

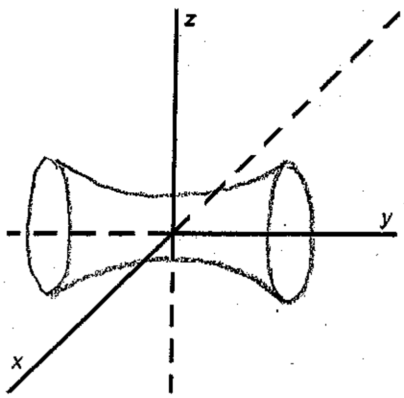
$\vec{T}\,'(t) = \frac{1}{5}\langle -8\sin 2t, 0, -8\cos 2t \rangle$

[d] Find the curvature.

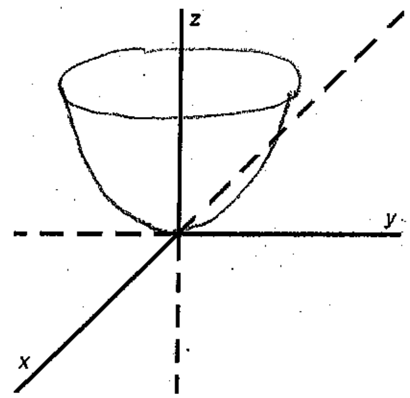
$$\kappa = \frac{\|\vec{T}\,'(t)\|}{\|\vec{r}\,'(t)\|} = \frac{\frac{1}{5}\sqrt{64\sin^2 2t + 64\cos^2 2t}}{5} = \frac{\frac{1}{5}\sqrt{64}}{5} = \frac{8}{25}$$

Name the surface for each equation below. Sketch the general shape and orientation of **three (3)** of the surfaces. **SCORE: _____ / 25 PTS**

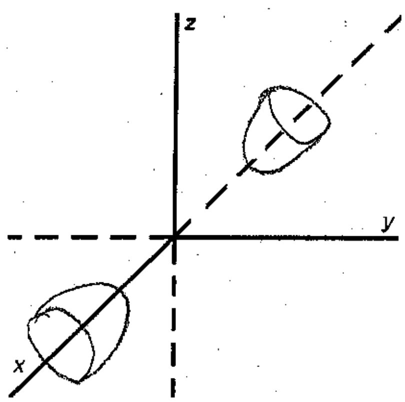
[a] $\frac{x^2}{9} - \frac{y^2}{4} + z^2 = 1$ NAME: HYPERBOLOID (1 SHEET)



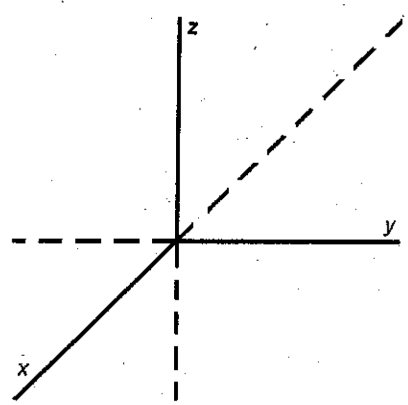
[b] $\frac{x^2}{4} + \frac{y^2}{9} - z = 0$ NAME: ELLIPTICAL PARABOLOID



[c] $\frac{x^2}{9} - \frac{y^2}{4} - z^2 = 1$ NAME: HYPERBOLOID (2 SHEETS)



[d] $\frac{x^2}{9} - \frac{y^2}{4} - z = 0$ NAME: HYPERBOLIC PARABOLOID



Find a vector function for the curve of intersection of the surfaces $z = \sqrt{x^2 + y^2}$ and $z = 1 + y$. **SCORE: _____ / 20 PTS**

$z^2 = x^2 + y^2 \qquad z^2 = 1 + 2y + y^2$

$x^2 + y^2 = 1 + 2y + y^2$

$x^2 = 1 + 2y$

$y = \frac{x^2 - 1}{2}$

$z = 1 + \frac{x^2 - 1}{2}$

$= \frac{x^2 + 1}{2}$

$\vec{r}(t) = \langle t, \frac{t^2 - 1}{2}, \frac{t^2 + 1}{2} \rangle$

Find the curvature of the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(1, 1, 1)$.

SCORE: _____ / 20 PTS

$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle \qquad \vec{r}'(1) = \langle 1, 2, 3 \rangle$

$\vec{r}''(t) = \langle 0, 2, 6t \rangle \qquad \vec{r}''(1) = \langle 0, 2, 6 \rangle$

$\vec{r}'(1) \times \vec{r}''(1) = \langle 6, -6, 2 \rangle$

$$\frac{\|\vec{r}'(1) \times \vec{r}''(1)\|}{\|\vec{r}'(1)\|^3} = \frac{\sqrt{36+36+4}}{(\sqrt{1+4+9})^3}$$

$$= \frac{\sqrt{76}}{\sqrt{14}^3} = \frac{2\sqrt{19}}{14\sqrt{14}} = \frac{\sqrt{19}}{7\sqrt{14}}$$

$$= \frac{\sqrt{266}}{98}$$

Find the curvature of $y = x^4$ at the point $(1, 1)$.

SCORE: _____ / 15 PTS

$$K(x) = \frac{12x^2}{[1+(4x^3)^2]^{\frac{3}{2}}}$$

$$K(1) = \frac{12}{(1+4^2)^{\frac{3}{2}}} = \frac{12}{17^{\frac{3}{2}}} = \frac{12}{17\sqrt{17}} = \frac{12\sqrt{17}}{289}$$

If $\vec{u}(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}''(t)]$, prove that $\vec{u}'(t) = \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$.

SCORE: _____ / 15 PTS

NOTE: You must give a general proof – you cannot assume that $\vec{r}(t)$ is any particular function.

$$\vec{u}'(t) = \vec{r}'(t) \cdot [\vec{r}'(t) \times \vec{r}''(t)] + \vec{r}(t) \cdot [\vec{r}''(t) \times \vec{r}''(t)] + \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$$

$$= [\vec{r}'(t) \times \vec{r}'(t)] \cdot \vec{r}''(t) + \vec{r}(t) \cdot \vec{0} + \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$$

$$= \vec{0} \cdot \vec{r}''(t) + \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$$

$$= \vec{r}(t) \cdot [\vec{r}'(t) \times \vec{r}'''(t)]$$